

# Spectral correlations in the crossover transition from a superposition of harmonic oscillators to the Gaussian unitary ensemble

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We compute the spectral correlation functions for the transition from a harmonic oscillator towards the Gaussian unitary ensemble (GUE). We use a variant of the supersymmetry method to obtain analytical results in a fast and elegant way. In contrast to certain related transitions, the  $k$ -point correlation function possesses the  $k \times k$  determinant structure of the GUE limit for the entire transition. The results are used to consider also the spectral correlations of a superposition of  $M$  transition spectra. Our results are nonperturbative and are valid for all values of the transition parameter. [S1063-651X(99)01801-2]

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## I. INTRODUCTION

Random matrix theory [1] is a powerful tool for the modeling of spectral fluctuation properties. Due to the general symmetry constraints, a time-reversal invariant system with conserved or broken rotation invariance is modeled by the Gaussian orthogonal or symplectic ensembles, respectively, while the Gaussian unitary ensemble (GUE) models the fluctuation properties of a system under broken time-reversal invariance. These ensembles are known to describe the generic fluctuation properties of chaotic quantum systems very accurately [2–5]. While numerical simulations of the ensuing matrix models usually pose no serious difficulties, the analytical calculations of the observables, i.e., the correlation functions, is generally a nontrivial task. In the case of the pure ensembles, Mehta and Dyson [1] solved the problem about 30 years ago by introducing the orthogonal polynomial method.

However, since a generic physical system has, classically, a mixed phase space, the spectral fluctuations of the corresponding quantum system will be in between the pure cases. The transition from preserved to broken time-reversal invariance was worked out by Mehta and Pandey [1]. The transitions of the spectral fluctuations in the case of gradually broken symmetries, i.e., quantum numbers were computed in Refs. [6] and [7]. Here we will focus on a system that undergoes a transition from regular to chaotic fluctuations. To model such a system, we write the  $N \times N$  random matrix representing the total Hamiltonian as a sum of a regular and a chaotic part

$$H(\alpha) = H^{(0)} + \alpha H^{(1)}, \quad (1)$$

where  $\alpha$  is the dimensionless transition parameter. The matrices  $H^{(1)}$  are drawn from a Gaussian ensemble with the probability density function  $P_N^{(1)}(H^{(1)})$ . Here we are mainly interested in the transition from a regular, equispaced spectrum to a chaotic one. This is a very important physical situation since many systems, particularly in nuclear and molecular physics, can be described as a chaos producing part coupled to a harmonic oscillator. Often it is necessary to give the regular part  $H^{(0)}$  a block structure that reflects the pres-

ence of symmetries. This was already realized by Porter and Rosenzweig [8], who investigated, experimentally and numerically, atomic spectra that contain various angular momenta and spin quantum numbers subject to different coupling schemes.

However, at the moment, we make no assumptions for the probability distribution  $P_N^{(0)}(H^{(0)})$  of the matrices  $H^{(0)}$ . The decomposition (1) can be justified for potential and billiard systems. Detailed numerical simulations for the transition of the fluctuations can be found in Ref. [9]. However, despite several attempts, full-fledged analytical discussions could only recently be performed for the case of broken time-reversal invariance in which the matrices  $H^{(1)}$  are drawn from the GUE. For a history of the studies devoted to these problems see Ref. [5]. Presently, there are the following techniques that make such calculations possible. Pandey [7] presented a certain construction of the solution of Dyson's Brownian motion model [3]. A related approach was more recently put forth by Forrester [10]. A very direct and compact technique for the GUE was constructed in Refs. [11,12]. It relies on a variant of the supersymmetry method [13,14] that was introduced in Ref. [15]. The enormous simplifications are due to the fact that supersymmetry can, loosely speaking, be viewed as the "irreducible representation" of random matrix theory, which becomes apparent in a different class of diffusion equations [12,5]. Recently, Brézin and Hikami [16] presented a third approach to derive similar integral representations.

In this paper we will apply the methods of Refs. [11,12] to the transition starting from a harmonic oscillator. Pandey [7] gave a formula for the two-level correlation function on the unfolded scale. Forrester [10] extended the result for the  $k$ -level correlation function. Here we have three goals. First, we will show that this result can be obtained very fast in a direct application of the general results of Refs. [11,12]. Second, we will present plots and a detailed discussion of the two-level correlation function. Third, we will go beyond the known results and study a block structure of  $H^{(0)}$  by considering the superposition of  $M$  transition spectra. After briefly sketching the method in Sec. II, we work out the crossover transition from one harmonic oscillator to the GUE in Sec.

III. In Sec. IV we study the superposition of  $M$  spectra. We summarize our results in Sec. V.

## II. TRANSITION ENSEMBLES

Before turning to the harmonic oscillator, we briefly summarize the general results. A detailed discussion can be found in Ref. [12]; see also Ref. [17].

As functions of the transition parameter  $\alpha$ , we wish to study the  $k$ -level correlation functions

$$R_k(x_1, \dots, x_k, \alpha) = \frac{1}{\pi^k} \int d[H^{(0)}] P_N^{(0)}(H^{(0)}) \int d[H^{(1)}] P_N^{(1)} \times (H^{(1)}) \prod_{p=1}^k \text{Im tr} \frac{1}{x_p^- - H(\alpha)} \quad (2)$$

depending on  $k$  energies  $x_p$ ,  $p=1, \dots, k$ , where the energies are given imaginary increments such that  $x_p^\pm = x_p \pm i\varepsilon$ . It is convenient to work with correlation functions  $\hat{R}_k(x_1, \dots, x_k, \alpha)$  that involve the full Green's functions, including real and imaginary parts. By studying different combinations of the signs of the imaginary parts of the Green's functions we can construct the physically interesting functions (2); see Ref. [12]. The correlation functions  $\hat{R}_k(x_1, \dots, x_k, \alpha)$  may be written as the derivatives

$$\hat{R}_k(x_1, \dots, x_k, \alpha) = \frac{1}{(2\pi)^k} \frac{\partial^k}{\prod_{p=1}^k \partial J_p} Z_k(x+J, \alpha)|_{J=0} \quad (3)$$

of the normalized generating functions  $Z_k(x+J, \alpha)$ . The energies and the source variables are ordered in the diagonal matrices  $x = \text{diag}(x_1, x_1, \dots, x_k, x_k)$  and  $J = \text{diag}(-J_1, +J_1, \dots, -J_k, +J_k)$ , respectively. The desired functions  $R_k(x_1, \dots, x_k, \alpha)$  can be derived from the generating function  $\mathfrak{J} Z_k$ , where the symbol  $\mathfrak{J}$  stands for the proper linear combination [12]. The average over the GUE is done by means of the standard techniques of the supersymmetry method [13,14], yielding

$$Z_k(x+J, \alpha) = \int d[H^{(0)}] P_N^{(0)}(H^{(0)}) \int d[\sigma] \times \exp\left(-\frac{1}{\alpha^2} \text{tr}_g \sigma^2\right) \det_g^{-1}[(x^\pm + J - \sigma) \otimes 1_N - 1_{2k} \otimes H^{(0)}], \quad (4)$$

where  $\det_g$  and  $\text{tr}_g$  are the graded determinant and trace, respectively. For other details of the derivation and notation, the reader is referred to Ref. [15]. In Eq. (4),  $\sigma$  denotes a  $2k \times 2k$  Hermitian supermatrix and  $1_N$  and  $1_{2k}$  are  $N \times N$  and  $2k \times 2k$  unit matrices, respectively.

To proceed, it is in our case advantageous to avoid the saddle point approximation of Refs. [13,14]. We shift the matrix  $x+J$  from the graded determinant to the graded probability density and the supermatrix  $\sigma$  is diagonalized according to  $\sigma = u^{-1} s u$ , where  $s = \text{diag}(s_{11}, i s_{12}, \dots, s_{k1}, i s_{k2})$ . The volume element can be rewritten as  $d[\sigma]$

$= B_k^2(s) d[s] d\mu(u)$  with  $B_k(s) = \det[1/(s_{p1} - i s_{q2})]_{p,q=1, \dots, k}$  the Jacobian, here referred to as Berezinian. The nontrivial integration over the unitary diagonalizing supergroup with its Haar measure  $d\mu(u)$  is the crucial step and can be performed with the supersymmetric extension [15] of the Harish-Chandra Itzykson Zuber integral [18]. Collecting everything we arrive at

$$Z_k(x+J, \alpha) = 1 - \eta(x+J) + \frac{1}{B_k(x+J)} \times \int G_k(s-x-J, \alpha) Z_k^{(0)}(s) B_k(s) d[s], \quad (5)$$

$$Z_k^{(0)}(x+J) = \int d[H_0] P_N^{(0)}(H_0) \times \det_g^{-1}[(x^\pm + J) \otimes 1_N - 1_{2k} \otimes H_0],$$

where the kernel resulting from the group integration is Gaussian and given by

$$G_k(s-r, \alpha) = \frac{1}{(\pi \alpha^2)^k} \exp\left(-\frac{1}{\alpha^2} \text{tr}_g (s-r)^2\right). \quad (6)$$

with  $r = x+J$ . The distribution  $1 - \eta(x+J)$  in Eq. (5) ensures the normalization  $Z_k(x, \alpha) = 1$  at  $J=0$ . This distribution is not important for any of the formulas to follow; see the discussion in Ref. [12]. The generating function  $Z_k(x+J, \alpha)$  satisfies an exact diffusion equation in the curved space of the eigenvalues of Hermitian supermatrices. Here  $t = \alpha^2/2$  is the diffusion time and the generating function  $Z_k^{(0)}(x+J)$  serves as the initial condition. This diffusion is the supersymmetric analog [12] of Dyson's Brownian motion [3].

The integration over  $s$  requires one to take a different type of boundary contribution [13,14,19] into account that do not occur in ordinary analysis. However, in Refs. [15,20] it was shown that we do not need to worry about them when calculating correlation functions of the type we are interested in here. Collecting everything, we obtain the  $k$ -level correlation functions

$$R_k(x_1, \dots, x_k, \alpha) = \frac{(-1)^k}{\pi^k} \int G_k(s-x, \alpha) \times \mathfrak{J} Z_k^{(0)}(s) B_k(s) d[s] \quad (7)$$

for nonzero  $\alpha$ . The case  $\alpha=0$  is trivial by construction.

As a last step it remains to unfold the correlation functions for large level number  $N$  by removing the dependence on the level density. We define new energies  $\xi_p = x_p/D$ ,  $p=1, \dots, k$ , in units of the mean level spacing  $D$ . The transition parameter has to be unfolded too,  $\lambda = \alpha/D$ , and was introduced by Pandey [21]. The  $k$ -level correlation functions on the unfolded scale  $X_k(\xi_1, \dots, \xi_k, \lambda) = \lim_{N \rightarrow \infty} D^k R_k(x_1, \dots, x_k, \alpha)$  are then generic, i.e., translation invariant over the spectrum. It is useful to unfold the integration variables  $s$  in Eq. (7) by making the rescaling  $s \rightarrow s/D$ . We arrive at

$$X_k(\xi_1, \dots, \xi_k, \lambda) = \frac{(-1)^k}{\pi^k} \int G_k(s - \xi, \lambda) \times \mathcal{J} z_k^{(0)}(s) B_k(s) d[s] \quad (8)$$

for nonzero  $\lambda$ , where the unfolded generating function of the arbitrary correlations is given by

$$z_k^{(0)}(s) = \lim_{N \rightarrow \infty} Z_k^{(0)}(Ds). \quad (9)$$

Hence we have expressed the unfolded  $k$ -level correlation function for the transition from arbitrary to GUE fluctuations as a  $2k$ -fold integral.

### III. TRANSITION FROM AN EQUISPACED SPECTRUM

All results derived so far are correct for arbitrary initial correlations  $R_k^{(0)}(x_1, \dots, x_k)$  or  $X_k^{(0)}(\xi_1, \dots, \xi_k)$ . We now apply them to the case of an equispaced spectrum. The harmonic oscillator probability density function reads

$$P_N^{(0)}(H^{(0)}) = \prod_{n=1}^N \delta(H_{nn}^{(0)} - \epsilon_n) \prod_{n>m} \delta(\text{Re } H_{nm}^{(0)}) \delta(\text{Im } H_{nm}^{(0)}), \quad (10)$$

where

$$\epsilon_n = \begin{cases} (n-1)D/2 + \delta D, & n=1,3,5, \dots, N \\ -nD/2 + \delta D, & n=2,4,6, \dots, N-1. \end{cases} \quad (11)$$

$D = 2\sqrt{2/N}$  is the mean level spacing and the number of levels  $N$  is odd. The spectrum is shifted by  $\delta$  and we require  $|\delta| < 1$ . We find from Eq. (5) for the generating function of the initial condition

$$Z_k^{(0)}(s) = \prod_{p=1}^k \frac{is_{p2} - \delta}{s_{p1}^{\pm} - \delta} \times \prod_{n=1}^{(N-1)/2} \frac{[1 - is_{p2}/(n+\delta)][1 + is_{p2}/(n-\delta)]}{[1 - is_{p1}^{\pm}/(n+\delta)][1 + is_{p1}^{\pm}/(n-\delta)]}. \quad (12)$$

According to Eq. (9) we evaluate the generating function on the unfolded scale in the limit  $N \rightarrow \infty$  and obtain

$$z_k^{(0)}(s) = \prod_{p=1}^k \frac{\sin \pi(is_{p2} - \delta)}{\sin \pi(s_{p1}^{\pm} - \delta)}. \quad (13)$$

The signs are determined by the choice of the sign of the imaginary increment in the Green's function. The correlation functions can be worked out by using the general result (8). The initial condition takes the form

$$\text{Im } z_k^{(0)}(s) = \prod_{p=1}^k \sin \pi(is_{p2} - \delta) \text{Im} \frac{1}{\sin \pi(s_{p1}^- - \delta)}. \quad (14)$$

We use the identity

$$\text{Im} \frac{1}{\sin \pi(s^- - \delta)} = \sum_{k=-\infty}^{\infty} (-1)^k \delta(s - k - \delta) \quad (15)$$

and insert Eqs. (6) and (14) into Eq. (8). Since the function  $B_k(s)$  is a determinant and since the generating function of the harmonic oscillator spectrum (14) and the Gaussian kernel (6) are products of  $2k$  factors, the  $k$ -point spectral correlation function may be written as a determinant [15]

$$X_k(\xi_1, \dots, \xi_k, \lambda) = \det[C(\xi_p, \xi_q, \lambda)]_{p,q=1, \dots, k}, \quad (16)$$

with

$$\begin{aligned} C(\xi_p, \xi_q, \lambda) &= -\frac{1}{\pi^2 \lambda^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{ds_{p1} ds_{q2}}{s_{p1} - is_{q2}} \\ &\times \exp\left(\frac{1}{\lambda^2} [(is_{q2} - \xi_q)^2 - (s_{p1} - \xi_p)^2]\right) \\ &\times \sin \pi(is_{q2} - \delta) \sum_{l=-\infty}^{+\infty} (-1)^l \\ &\times \delta(s_{p1} - l - \delta) \\ &= \frac{1}{\pi^2 \lambda^2} \sum_{l=-\infty}^{+\infty} (-1)^l \exp\left(-\frac{(\xi_p - \delta - l)^2}{\lambda^2}\right) \\ &\times \int_{-\infty}^{+\infty} ds_{q2} \frac{\sin \pi(is_{q2} - \delta)}{s_{q2} + i(l + \delta)} \\ &\times \exp\left(-\frac{(s_{q2} + i\xi_q)^2}{\lambda^2}\right). \end{aligned} \quad (17)$$

The remaining integration may be done as follows. First, we translate the path of integration about  $+i\delta$  that does not change the value of the integral. Then we write the denominator of the integrand as a Laplace transform and perform the integrations by means of [22]

$$\begin{aligned} &\int_{-\infty}^{\infty} ds \frac{\sinh \pi s}{s + il} \exp\left(-\frac{(s + i(\xi - \delta))^2}{\lambda^2}\right) \\ &= -\pi(-1)^l \exp\left(\frac{(\xi - \delta - l)^2}{\lambda^2}\right) \\ &\times \text{Im} \text{erf}\left(\frac{l - (\xi - \delta)}{\lambda} - \text{sgn}(l) - i\frac{\pi}{2}\lambda\right). \end{aligned} \quad (18)$$

This yields (erf denotes the error function as defined in Ref. [22])

$$\begin{aligned}
C(\xi_p, \xi_q, \lambda) &= -\frac{1}{\pi\lambda^2} \exp\left(-\frac{(\xi_p - \delta)^2 - (\xi_q - \delta)^2}{\lambda^2}\right) \\
&\times \sum_{l=-\infty}^{\infty} \exp\left(-\frac{2}{\lambda^2}(\xi_q - \xi_p)l\right) \\
&\times \text{Im erf}\left(\frac{l - (\xi_q - \delta)}{\lambda} \text{sgn}(l) - i\frac{\pi}{2}\lambda\right).
\end{aligned} \tag{19}$$

To proceed, we use the definition of the error function and write it as the integral of a Gaussian. Taking its imaginary part and writing the just introduced Gaussian as a Fourier transform yields

$$\begin{aligned}
C(\xi_p, \xi_q, \lambda) &= \frac{1}{4\pi} \int_{-1}^1 dt \exp\left(\frac{\pi^2 \lambda^2 t^2}{4}\right) \\
&\times \int_{-\infty}^{\infty} ds \exp\left(-\frac{\lambda^2 s^2}{4}\right) \\
&\times \sum_{l=-\infty}^{\infty} \cos[\pi(\xi_q - \delta - l)t] \\
&\times \cos[(\xi_p - \delta - l)s].
\end{aligned} \tag{20}$$

We use Poisson's sum formula and perform the remaining integrations. The final result displays  $C(\xi_p, \xi_q, \lambda)$  as a Fourier series

$$\begin{aligned}
C(\xi_p, \xi_q, \lambda) &= \sum_{l=-\infty}^{\infty} \exp(-\pi^2 \lambda^2 l^2) \text{Re} \left( \exp[i2\pi l(\xi_p - \delta)] \right. \\
&\times \left. \frac{\sinh[\pi^2 \lambda^2 l + i\pi(\xi_q - \xi_p)]}{\pi^2 \lambda^2 l + i\pi(\xi_q - \xi_p)} \right),
\end{aligned} \tag{21}$$

which inserted in Eq. (16) completes our calculation. Two interesting limits may be considered. In the limit of vanishing transition parameter one obtains

$$\lim_{\lambda \rightarrow 0} C(\xi_p, \xi_q, \lambda) = \frac{\sin \pi(\xi_q - \xi_p)}{\pi(\xi_q - \xi_p)} \sum_{l=-\infty}^{\infty} \delta(\xi_p - \delta - l), \tag{22}$$

which generates the spectral correlations of the harmonic oscillator. The opposite limit of infinite transition parameter yields the GUE spectral correlations, i.e.,

$$\lim_{\lambda \rightarrow \infty} C(\xi_p, \xi_q, \lambda) = \frac{\sin \pi(\xi_p - \xi_q)}{\pi(\xi_p - \xi_q)}. \tag{23}$$

Plots of the level density

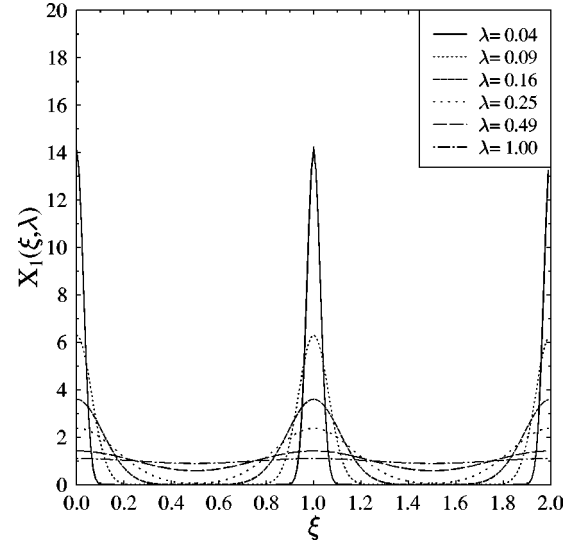


FIG. 1. Level density  $X_1(\xi, \lambda)$  for different values of the transition parameter  $\lambda$ .

$$\begin{aligned}
X_1(\xi, \lambda) &= C(\xi, \xi, \lambda) \\
&= \sum_{l=-\infty}^{\infty} \exp(-\pi^2 \lambda^2 l^2) \frac{\sinh \pi^2 \lambda^2 l}{\pi^2 \lambda^2 l} \\
&\times \cos 2\pi l(\xi - \delta)
\end{aligned} \tag{24}$$

are shown for different values of the transition parameter  $\lambda$  in Fig. 1. Note that very small values of the transition parameter  $\lambda$  already cause a considerable broadening of the  $\delta$  peaks of the original harmonic oscillator spectrum. It has been observed in almost all crossover transitions that, on scales of a few mean level spacings, the GUE limit is reached when  $\lambda$  is of the order of unity; see the review in Ref. [5]. Figures 2 and 3 show that this is also true for the two-point function

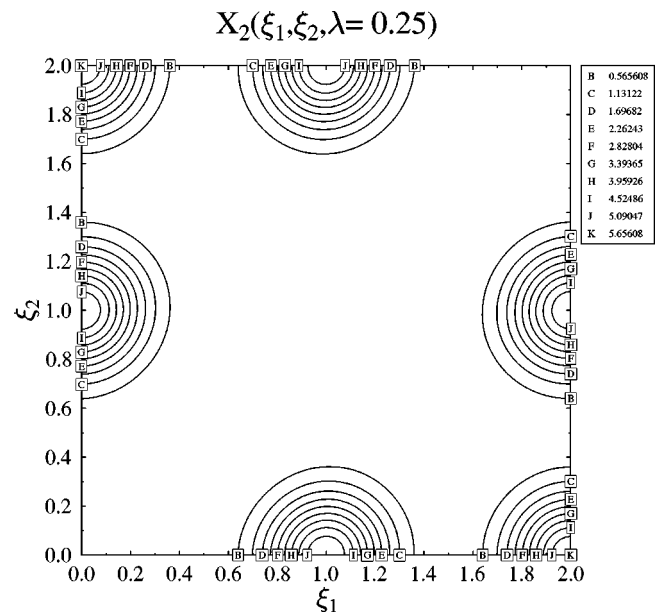


FIG. 2. Contour plot of  $X_2(\xi_1, \xi_2, \lambda = 0.25)$ .

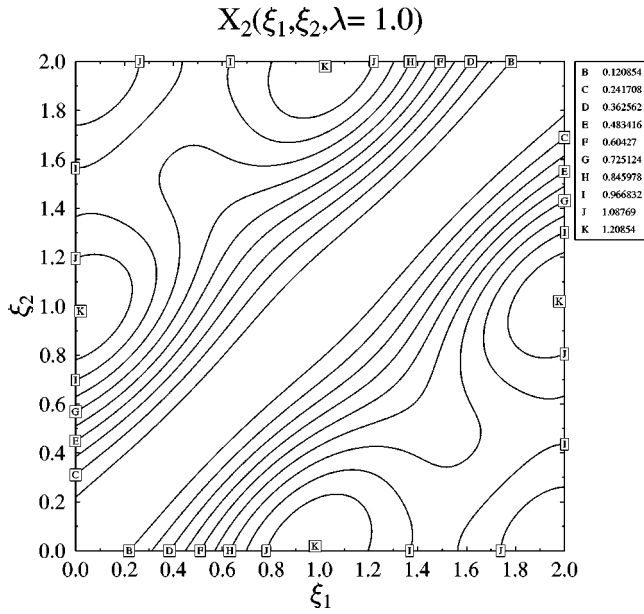


FIG. 3. Contour plot of  $X_2(\xi_1, \xi_2, \lambda = 1.0)$ .

$$X_2(\xi_1, \xi_2, \lambda) = X_1(\xi_1, \lambda)X_1(\xi_2, \lambda) - C(\xi_1, \xi_2, \lambda)C(\xi_2, \xi_1, \lambda). \quad (25)$$

The  $\delta$  peaks at integer values  $\xi_1 \neq \xi_2$  are broadened for a small value of the transition parameter  $\lambda$  and the GUE correlations are approached fast. Note also that the two-point function (25) becomes translationally invariant (i.e., it depends only on the distance  $\xi_1 - \xi_2$ ) if averaging over  $\delta$  is done [10].

#### IV. SUPERPOSITION OF SPECTRA

We now consider a superposition of independent, noninteracting spectra as the initial condition. This is equivalent to saying that there is a symmetry that allows one to write the initial condition in the form  $H^{(0)} = \text{diag}(H_1^{(0)}, \dots, H_M^{(0)})$ , where each of the  $N_m \times N_m$  matrices  $H_m^{(0)}$ ,  $m = 1, \dots, M$ , is drawn from an independent ensemble. We have  $\sum_{m=1}^M N_m = N$ , the total dimension, and  $\sum_{m=1}^M D_m^{-1} = D^{-1}$ , where  $D_m$  is the mean level spacing in the  $m$ th ensemble. The quantities  $g_m = D/D_m$  are referred to as fractional level densities. If all  $N_m$  are equal, we have  $g_m = 1/M$ . Symmetries that lead to such models are frequently found in nuclear, atomic, and molecular physics [8,5].

For given fluctuations of every subensemble with index  $m$ , the fluctuations properties of the superposition can be worked out in a rather straightforward way. In the case of the nearest neighbor spacing distribution, the result is given in the article by Porter and Rosenzweig [8]. More general discussion may be found in Ref. [1] and in the review [5].

Here we address the case that all matrices  $H_m^{(0)}$  are drawn from harmonic oscillators. In discussing the influence of a chaotic admixture on this initial condition, two different scenarios are of interest: The chaotic admixture can either preserve the symmetry or break it. In the first scenario, we have to add a GUE matrix to *every* matrix  $H_m^{(0)}$ . Thus we still have a superposition of independent spectra and the total

matrix  $H(\alpha)$  still has the block structure, i.e.,  $H(\alpha) = \text{diag}[H_1(\alpha), \dots, H_M(\alpha)]$ .

In the second scenario, we add one  $N \times N$  GUE matrix to the block diagonal matrix  $H^{(0)}$ . Thus, for a nonzero transition parameter, the total matrix  $H(\alpha)$  has no block structure. This scenario is in the spirit of the Porter-Rosenzweig model [8]. For  $M = 2$  such a symmetry breaking was investigated in Refs. [6] and [7]. In these studies, however, and in contrast to the present one, the initial condition was also chaotic. We discuss the first scenario in Sec. IV A, before we briefly turn to the second one in Sec. IV B.

#### A. Symmetry preserving case

Each of the block matrices is a harmonic oscillator, coupled to a GUE. For simplicity, we assume that all blocks matrices shall have the same dimension  $N_m$  and the same coupling to the GUE. Moreover, to lift the degeneracies, the  $m$ th oscillator spectrum is chosen with an energy shift  $\delta_m = m/M$  [cf. Eq. (11)]. We are interested in the unfolded two-point correlation function

$$\begin{aligned} X_2^{(M)}(\xi_1, \xi_2, \lambda) &= \sum_{m=1}^M g_m^2 X_{2,m}(g_m \xi_1, g_m \xi_2, g_m \lambda) \\ &+ \sum_{n,m=1}^M g_m g_n X_{1,m}(g_m \xi_1, g_m \lambda) \\ &\times X_{1,n}(g_n \xi_2, g_n \lambda) - \sum_{m=1}^M g_m^2 \\ &\times X_{1,m}(g_m \xi_1, g_m \lambda) X_{1,m}(g_m \xi_2, g_m \lambda), \end{aligned} \quad (26)$$

where  $g_m = 1/M$  is the fractional density of each block matrix. To motivate this equation, we notice that  $X_2^{(M)}(\xi_1, \xi_2, \lambda)$  is the probability density of finding a level at  $\xi_1$  and another one at  $\xi_2$ . The levels can belong either to the same symmetry sector [first line of Eq. (26)] or to different symmetry sectors [second and third lines of Eq. (26)]. Since the second line also includes correlations within one symmetry sector (these have been accounted for in the first line), the third line must be subtracted. We emphasize once more that the functions  $X_{1,m}(g_m \xi_1, g_m \lambda)$  are not unity in the case of a harmonic oscillator.

We now insert the two-point function (25) with the appropriate energy shifts into Eq. (26) and interchange the summation over  $m$  with the summation over  $k, l$ . For  $M > 1$ , this yields expressions such as

$$\begin{aligned} &\sum_{m=1}^M \cos 2\pi \frac{k}{M} (\xi_1 - m) \cos 2\pi \frac{l}{M} (\xi_2 - m) \\ &= \frac{M}{2} (\delta_{k,l} + \delta_{k,-l}) \cos 2\pi \frac{k}{M} (\xi_1 - \xi_2), \end{aligned} \quad (27)$$

which help to perform one more summation. The final result reads for  $M > 1$

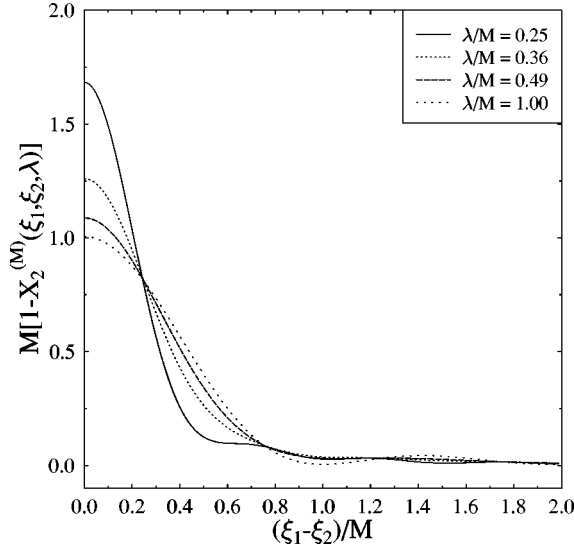


FIG. 4. Plot of  $M[1 - X_2^{(M)}(\xi_1, \xi_2, \lambda)]$  for different values of the transition parameter  $\lambda$ .

$$\begin{aligned}
 X_2^{(M)}(\xi_1, \xi_2, \lambda) &= 1 - \frac{1}{M} \sum_{k=-\infty}^{\infty} \left| \frac{\sinh[\pi^2 \lambda^2 k / M^2 + i \pi (\xi_1 - \xi_2) / M]}{\pi^2 \lambda^2 k / M^2 + i \pi (\xi_1 - \xi_2) / M} \right|^2 \\
 &\quad \times \exp\left(-2 \pi^2 \lambda^2 \frac{k^2}{M^2}\right) \cos 2 \pi \frac{k}{M} (\xi_1 - \xi_2). \quad (28)
 \end{aligned}$$

In the limit of vanishing  $\lambda$  we find

$$\lim_{\lambda \rightarrow 0} X_2^{(M)}(\xi_1, \xi_2, \lambda) = 1 - \delta(\xi_1 - \xi_2) \quad (29)$$

and in the limit of infinite transition parameter we recover the two-point correlation function for a superposition of  $M$  GUE's

$$\lim_{\lambda \rightarrow \infty} X_2^{(M)}(\xi_1, \xi_2, \lambda) = 1 - \frac{1}{M} \left( \frac{\sin \pi (\xi_1 - \xi_2) / M}{\pi (\xi_1 - \xi_2) / M} \right)^2. \quad (30)$$

For large  $M$  and nonzero transition parameter  $\lambda$ , the two-point correlation function approaches the value one. Thus the spectrum becomes completely uncorrelated and exhibits the same correlations as the (randomly generated) Poisson spectrum. We also note that the two-point correlation function (28) depends only on the difference  $(\xi_1 - \xi_2)$  of its arguments. This translation invariance results from the assumption that the  $m$ th spectrum is shifted by  $m/M$  mean level spacings. Thus any deviation from translation invariance gives a hint to clustering or degeneracy of levels belonging to different symmetry sectors. Figure 4 shows the plot of  $M[1 - X_2^{(M)}(\xi_1, \xi_2, \lambda)]$  for various values of the coupling  $\lambda$ . This function depends only on the ratios  $\lambda/M$  and  $(\xi_1 - \xi_2)/M$ .

## B. Symmetry breaking case

We turn to the second scenario defined above. We do this briefly because we mainly want to show the very different structure of the result. In general, the initial condition on the original scale takes the form

$$Z_k^{(0)}(s) = \prod_{m=1}^M Z_{km}^{(0)}(s), \quad (31)$$

where the initial conditions  $Z_{km}^{(0)}(s)$  are still completely arbitrary. On the unfolded scale, we have

$$z_k^{(0)}(s) = \prod_{m=1}^M z_{km}^{(0)}(g_m s). \quad (32)$$

These formulas have to be inserted into Eqs. (7) and (8), respectively, to obtain the correlation functions.

If all  $M$  initial ensembles are chosen as harmonic oscillators, we find from Eq. (13) on the unfolded scale

$$z_k^{(0)}(s) = \prod_{m=1}^M \prod_{p=1}^k \frac{\sin \pi (g_m i s p_2 - \delta_m)}{\sin \pi (g_m s_{p_1}^\pm - \delta_m)}. \quad (33)$$

Since this is still a product, the determinant structure (16) of the  $k$ -point correlation functions is still preserved. The functions

$$\begin{aligned}
 C(\xi_p, \xi_q, \lambda) &= -\frac{1}{\pi^2 \lambda^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{ds_{p_1} ds_{q_2}}{s_{p_1} - i s_{q_2}} \\
 &\quad \times \exp\left(\frac{1}{\lambda^2} [(i s_{q_2} - \xi_q)^2 - (s_{p_1} - \xi_p)^2]\right) \\
 &\quad \times \prod_{m=1}^M \sin \pi (g_m i s_{q_2} - \delta_m) \\
 &\quad \times \text{Im} \prod_{m'=1}^M \frac{1}{\sin \pi (g_{m'} s_{q_1} - \delta_{m'})} \quad (34)
 \end{aligned}$$

are the entries of this determinant. For general  $g_m$  and  $\delta_m$ , this result is not easily amenable to simplifications. In some special cases, however, the product over  $m$  in Eq. (33) can be performed. In particular, for  $g_m = 1/M$  and  $\delta_m = m/M$  we obtain

$$z_k^{(0)}(s) = \prod_{p=1}^k \frac{\sin \pi i s p_2}{\sin \pi s_{p_1}^\pm} \quad (35)$$

by using standard results for trigonometric functions. This is precisely Eq. (13) for  $\delta=0$ . Thus we recover the results of Sec. III because the case discussed here corresponds to the coupling of one oscillator, with the spacing on the original scale multiplied by  $1/M$ , to one GUE matrix.

## V. SUMMARY

We have computed all spectral correlation functions for the transition from a harmonic oscillator to the GUE. We

have used these results to compute the two-point correlation function for a system that undergoes a crossover transition to a GUE starting from a superposition of independent oscillator spectra. We discussed chaotic admixtures to an initial block structure that either preserve or break this symmetry. These ideas and techniques can also be used for other block diagonal initial conditions with different individual statistics.

Applying a variant of the supersymmetry technique, we gave an elegant and very straightforward derivation of these results.

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